NAG Toolbox for MATLAB

f04ff

1 Purpose

f04ff solves the equations $Tx = b$, where T is a real symmetric positive-definite Toeplitz matrix.

2 Syntax

```
[x, p, ifail] = f04ff(t, b, wantp, 'n', n)
```

3 Description

f04ff solves the equations

$$Tx = b,$$

where T is the n by n symmetric positive-definite Toeplitz matrix

$$T = \begin{pmatrix} \tau_0 & \tau_1 & \tau_2 & \dots & \tau_{n-1} \\ \tau_1 & \tau_0 & \tau_1 & \dots & \tau_{n-2} \\ \tau_2 & \tau_1 & \tau_0 & \dots & \tau_{n-3} \\ . & . & . & & . \\ \tau_{n-1} & \tau_{n-2} & \tau_{n-3} & \dots & \tau_0 \end{pmatrix}$$

and b is an n element vector.

The function uses the method of Levinson (see Levinson 1947 and Golub and Van Loan 1996). Optionally, the reflection coefficients for each step may also be returned.

4 References

Bunch J R 1985 Stability of methods for solving Toeplitz systems of equations *SIAM J. Sci. Statist. Comput.* **6** 349–364

Bunch J R 1987 The weak and strong stability of algorithms in numerical linear algebra *Linear Algebra Appl.* **88/89** 49–66

Cybenko G 1980 The numerical stability of the Levinson–Durbin algorithm for Toeplitz systems of equations *SIAM J. Sci. Statist. Comput.* **1** 303–319

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Levinson N 1947 The Weiner RMS error criterion in filter design and prediction *J. Math. Phys.* **25** 261–278

5 Parameters

5.1 Compulsory Input Parameters

1: $\mathbf{t}(\mathbf{0} : *)$ **– double array**

Note: the dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

$\mathbf{t}(i)$ must contain the value τ_i, for $i = 0, 1, \dots, \mathbf{n} - 1$.

Constraint: $\mathbf{t}(0) > 0.0$. Note that if this is not true, then the Toeplitz matrix cannot be positive-definite.

2: **b(∗) – double array**

Note: the dimension of the array **b** must be at least $\max(1,\mathbf{n})$.

The right-hand side vector b.

3: **wantp – logical scalar**

Must be set to **true** if the reflection coefficients are required, and must be set to **false** otherwise.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **t** The dimension of the array **b**.

The order of the Toeplitz matrix T.

Constraint: $\mathbf{n} \geq 0$. When $\mathbf{n} = 0$, then an immediate return is effected.

5.3 Input Parameters Omitted from the MATLAB Interface

work

5.4 Output Parameters

1: **x(∗) – double array**

Note: the dimension of the array **x** must be at least $\max(1,\mathbf{n})$.

The solution vector x.

2: **p(∗) – double array**

Note: the dimension of the array **p** must be at least $\max(1,\mathbf{n}-1)$ if **wantp** = **true**, and at least 1 otherwise.

With **wantp** as **true**, the ith element of **p** contains the reflection coefficient, p_i, for the ith step, for $i = 1, 2, \ldots, \mathbf{n}-1$. (See Section 8.) If **wantp** is **false**, then **p** is not referenced.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: f04ff may return useful information for one or more of the following detected errors or warnings.

ifail $= -1$

On entry, $\mathbf{n} < 0$,
or $\mathbf{t}(0) \leq 0.0$.

ifail > 0

The principal minor of order **ifail** of the Toeplitz matrix is not positive-definite to working accuracy. The first (**ifail** -1) elements of **x** return the solution of the equations

$$T_{\mathbf{ifail}-1}x = \left(b_1, b_2, \ldots, b_{\mathbf{ifail}-1}\right)^{\mathrm{T}},$$

where T_k is the kth principal minor of T.

7 Accuracy

The computed solution of the equations certainly satisfies

$$r = Tx - b,$$

where $\|r\|$ is approximately bounded by

$$\|r\| \leq c\epsilon C(T),$$

c being a modest function of n, ϵ being the ***machine precision*** and $C(T)$ being the condition number of T with respect to inversion. This bound is almost certainly pessimistic, but it seems unlikely that the method of Levinson is backward stable, so caution should be exercised when T is ill-conditioned. The following bound on T^{-1} holds:

$$\max\left(\frac{1}{\prod_{i=1}^{n-1}(1-p_i^2)}, \frac{1}{\prod_{i=1}^{n-1}(1-p_i)}\right) \leq \left\|T^{-1}\right\|_1 \leq \prod_{i=1}^{n-1}\left(\frac{1+|p_i|}{1-|p_i|}\right).$$

(See Golub and Van Loan 1996.) The norm of T^{-1} may also be estimated using function f04yc. For further information on stability issues see Bunch 1985, Bunch 1987, Cybenko 1980 and Golub and Van Loan 1996.

8 Further Comments

The number of floating-point operations used by f04ff is approximately $4n^2$.

If y_i is the solution of the equations

$$T_i y_i = -(\tau_1 \tau_2 \ldots \tau_i)^{\mathrm{T}},$$

then the partial correlation coefficient p_i is defined as the ith element of y_i.

9 Example

```
t = [4;
     3;
     2;
     1];
b = [1;
     1;
     1;
     1];
wantp = true;
[x, p, ifail] = f04ff(t, b, wantp)
```

```
x =
    0.2000
   -0.0000
    0.0000
    0.2000
p =
   -0.7500
    0.1429
    0.1667
ifail =
           0
```